Pirani gage and, unlike ionization gages, it is immune to chemical decomposition of the air, which begins about 90 km. Recently, Electro-Optical Systems, Inc. has investigated the adaptation of a solid-state strain gage as a pressure transducer. Deflection of the diaphragm causes a strain in the silicon crystal elements fastened to its surface. The piezoresistive effect induced in the crystal by the strain is a measure of the pressure force, causing the diaphragm to deflect.

Reference

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Manned Vehicles as Solids with Translating Particles: I

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Men moving within a space vehicle are extensible objects; furthermore, the motion of the vehicle feeds back on their ability to execute a desired path. It is assumed that they are small relative to the vehicle, whereas the latter is somewhat quiescent, but in the purely mathematical situation, there is no restriction placed on spin. We thus study the formal problem of a spinning rigid body within which a given number of mass points slide along grooves in a prescribed manner. It is desired to determine the effect of these particle motions on the rotation of the body when no external torques exist.

The equations of motion for a system of interacting rigid bodies are first derived from basic principles. An argument similar to that of Abzug[1] is given, but the final result is cast in a form different from his. By referencing all translations to the composite center of mass, we get an expression with no reference to inertial space displacement. In this way we construct inertia dyadics that depend only on variables defined onboard the body. The form of these dyadics suggests the kinds of particle constraints that can lead to equations similar to free-body motion. In particular, the case of a body of revolution, in which all particles remain on the symmetry axis, has a straightforward solution: the motion of an axially symmetric body with constant mass and time-varying longitudinal moment of inertia. We end the text with a detailed example for the rotation of a symmetric body having one particle whose relative motion is sinusoidal.

General Equation

Let an indefinite number of rigid bodies P^i be moving about in space, and choose one as the main body with a reference frame B^0 attached to it. Let $\boldsymbol{\beta}^i$ be the displacement of the mass center (c.m.) of P^i from B^0, $\boldsymbol{\omega}_0{}^i$ be the angular velocity of P^i relative to B^0, and $\mathbf{I}^i$ be the inertia dyadic of P^i about its own c.m. The equation of translation for P^i relative to space-fixed axes S is then

$$m^i \frac{d^2}{dt^2}(\boldsymbol{\sigma} + \boldsymbol{\beta}^i) = \mathbf{f}^i + \sum_{j\neq i} \mathbf{f}^{ij}$$

where $\boldsymbol{\sigma}$ is the displacement of the origin of B^0 from the origin of S, whereas $\mathbf{f}^i$ and $\mathbf{f}^{ij}$ are, respectively, the total external force and the force due to P^j acting on P^i. The equation of rotation for P^i, relative to space-fixed axes S, is as follows:

$$\frac{d}{dt}\left[\mathbf{I}^i\cdot(\boldsymbol{\omega} + \boldsymbol{\omega}_0{}^i) + (\boldsymbol{\sigma} + \boldsymbol{\beta}^i) \times m^i \frac{d}{dt}(\boldsymbol{\sigma} + \boldsymbol{\beta}^i)\right] = \mathbf{l}^i + (\boldsymbol{\sigma} + \boldsymbol{\beta}^i) \times \mathbf{f}^i + \sum_{j\neq i}[\mathbf{l}^{ij} + (\boldsymbol{\sigma} + \boldsymbol{\beta}^i) \times \mathbf{f}^{ij}]$$

where $\boldsymbol{\omega}$ is the angular velocity of B^0 relative to S, whereas $\mathbf{l}^i$ and $\mathbf{l}^{ij}$ are, respectively, the total external torque and the torque due to P^j acting on P^i about its center of mass. It must be true from both equations that

$$\sum_{i=0}^{n}\sum_{j\neq i} \mathbf{f}^{ij} = 0$$

$$\sum_{i=0}^{n}\sum_{j\neq i} [\mathbf{l}^{ij} + (\boldsymbol{\sigma} + \boldsymbol{\beta}^i) \times \mathbf{f}^{ij}] = 0$$

for if not, and all $\mathbf{f}^i$, $\mathbf{l}^i$ were zero, then the linear and angular momenta of the system relative to S would not be constant. If now, $\mathbf{c}$ is the center of mass of the system in terms of S, then

$$\boldsymbol{\sigma} + \boldsymbol{\beta}^i = \mathbf{c} + \boldsymbol{\beta}^i - \frac{1}{m}\sum_{j=0}^{n} m^j\boldsymbol{\beta}^j = \mathbf{c} + \mathbf{r}^i$$

where m is the total mass and $\mathbf{r}^i$ is the displacement of the P^i center of mass from $\mathbf{c}$. We next rewrite two expressions from the rotation equations by summing over the bodies

$$\sum_{i=0}^{n}\frac{d}{dt}\left[(\boldsymbol{\sigma} + \boldsymbol{\beta}^i) \times m^i \frac{d}{dt}(\boldsymbol{\sigma} + \boldsymbol{\beta}^i)\right] = \mathbf{c} \times m\frac{d^2}{dt^2}\mathbf{c} + \sum_{i=0}^{n}\mathbf{r}^i \times m^i\frac{d^2}{dt^2}\mathbf{r}^i$$

$$\sum_{i=0}^{n}(\boldsymbol{\sigma} + \boldsymbol{\beta}^i) \times \mathbf{f}^i = \mathbf{c} \times \mathbf{f} + \sum_{i=0}^{n}\mathbf{r}^i \times \mathbf{f}^i$$

where $\mathbf{f}$ is the sum of $\mathbf{f}^i$. Finally, on summing all the equations of translation, cross multiplying on the left by $\mathbf{c}$, and subtracting from the summed rotation equations, one gets

$$\sum_{i=0}^{n}\frac{d}{dt}[\mathbf{I}^i\cdot(\boldsymbol{\omega} + \boldsymbol{\omega}_0{}^i)] + \sum_{i=0}^{n}\mathbf{r}^i \times m^i\frac{d^2}{dt^2}\mathbf{r}^i = \sum_{i=0}^{n}\mathbf{l}^i + \sum_{i=0}^{n}\mathbf{r}^i \times \mathbf{f}^i$$

so that retaining just the main body, reducing the rest to points, and setting external influences to zero

$$\frac{d}{dt}(\mathbf{I}\cdot\boldsymbol{\omega}) + \sum_{i=0}^{n}\mathbf{r}^i \times m^i\frac{d^2}{dt^2}\mathbf{r}^i = 0 \qquad (1)$$

with $\mathbf{I} = \mathbf{I}^0$.

With respect to the body,

$$d(\mathbf{I}\cdot\boldsymbol{\omega})/dt = \mathbf{I}\cdot\dot{\boldsymbol{\omega}} + \boldsymbol{\omega} \times \mathbf{I}\cdot\boldsymbol{\omega} \qquad (2a)$$

$$d^2\mathbf{r}^i/dt^2 = \ddot{\mathbf{r}}^i + 2\boldsymbol{\omega} \times \dot{\mathbf{r}}^i + \dot{\boldsymbol{\omega}} \times \mathbf{r}^i + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r}^i) \qquad (2b)$$

Now we have as follows:

$$\mathbf{r}^i \times (2\boldsymbol{\omega} \times \dot{\mathbf{r}}^i) = -2\mathbf{r}^i \times (\dot{\mathbf{r}}^i \times \boldsymbol{\omega}) = -2(\mathbf{R}^i\cdot\dot{\mathbf{R}}^i)\cdot\boldsymbol{\omega}$$

where

$$\mathbf{R}\cdot\dot{\mathbf{R}} = -(\mathbf{r}\cdot\dot{\mathbf{r}})\mathbf{E} + t\mathbf{r}$$

and $\mathbf{E}$ is the idemfactor. Similarly,

$$\mathbf{r}^i \times (\dot{\boldsymbol{\omega}} \times \mathbf{r}^i) = -(\mathbf{R}^i\cdot\mathbf{R}^i)\cdot\dot{\boldsymbol{\omega}}$$

while ($\boldsymbol{\Omega}$ is formed similar to $\mathbf{R}$)

$$\mathbf{r}^i \times [\boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r}^i)] = \mathbf{r}^i \times (\boldsymbol{\Omega}\cdot\boldsymbol{\Omega})\cdot\mathbf{r}^i = \mathbf{r}^i \times \boldsymbol{\omega}\boldsymbol{\omega}\cdot\mathbf{r}^i = -\boldsymbol{\omega} \times \mathbf{r}^i\mathbf{r}^i\cdot\boldsymbol{\omega} = -\boldsymbol{\omega} \times (\mathbf{R}^i\cdot\mathbf{R}^i)\cdot\boldsymbol{\omega}$$

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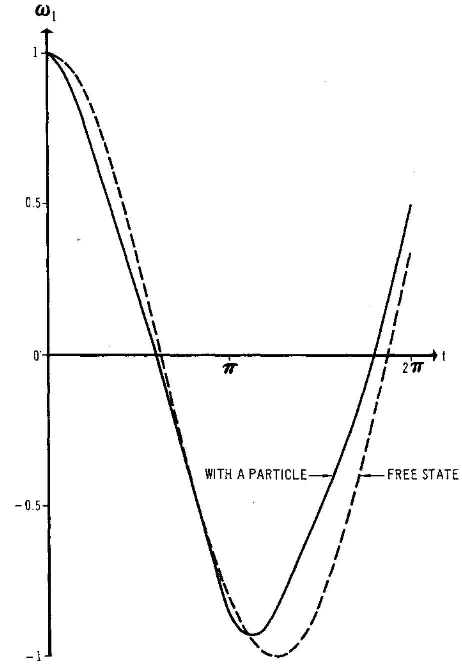

Fig. 1

In terms of the foregoing, we rewrite (1) using (2)

$$\left(\mathbf{I} - \sum_{i=0}^{n} m^i\mathbf{R}^i\cdot\mathbf{R}^i\right)\cdot\dot{\boldsymbol{\omega}} + \boldsymbol{\omega}\times\left(\mathbf{I} - \sum_{i=0}^{n} m^i\mathbf{R}^i\cdot\mathbf{R}^i\right)\cdot\boldsymbol{\omega} - 2\left(\sum_{i=0}^{n} m^i\mathbf{R}^i\cdot\dot{\mathbf{R}}^i\right)\cdot\boldsymbol{\omega} + \sum_{i=0}^{n} m^i\mathbf{r}^i\times\ddot{\mathbf{r}}^i = \mathbf{0} \quad (3)$$

Equation (3) is of the general form

$$\mathbf{A}(t)\cdot\dot{\boldsymbol{\omega}} + \boldsymbol{\omega}\times\mathbf{A}(t)\cdot\boldsymbol{\omega} + \mathbf{B}(t)\cdot\boldsymbol{\omega} + \mathbf{c}(t) = \mathbf{0} \quad (4)$$

Special Integral

If the body is dynamically of revolution and the particles are constrained to move along the symmetry axis, i.e.,

$$\mathbf{r}^i = r_3{}^i\begin{pmatrix}0\\0\\1\end{pmatrix}$$

for all i, then we get a solvable form for the dyadics in (3) similar to that of free-body motion:

$$\mathbf{R}^i\cdot\mathbf{R}^i = -[r_3{}^i(t)]^2\begin{pmatrix}1 & & \\ & 1 & \\ & & 0\end{pmatrix}$$

$$\mathbf{R}^i\cdot\dot{\mathbf{R}}^i = -r_3{}^i(t)\dot{r}_3{}^i(t)\begin{pmatrix}1 & & \\ & 1 & \\ & & 0\end{pmatrix}$$

so that, with

$$\mathbf{I} = \begin{pmatrix}I_0 & & \\ & I_0 & \\ & & I_3\end{pmatrix}$$

one has for A, B (diagonal) such that $A_{33}(t) = I_3$, $B_{33}(t) = 0$, $\mathbf{c}(t) = \mathbf{0}$, and

$$A_{11}(t) = A_{22}(t) = A_{00}(t) = I_0 + \sum_{i=0}^{n} m_i[r_3{}^i(t)]^2$$

$$B_{11}(t) = B_{22}(t) = B_{00}(t) = 2\sum_{i=0}^{n} m^i r_3{}^i(t)\dot{r}_3{}^i(t)$$

This leads to Eq. (5) in scalar form

$$A_{00}(t)\dot{\omega}_i + [A_{33}(t) - A_{00}(t)]\omega_3\omega_2 + B_{00}(t)\omega_1 = 0 \quad (5a)$$

$$A_{00}(t)\dot{\omega}_2 - [A_3(t) - A_{00}(t)]\omega_3\omega_1 + B_{00}(t)\omega_2 = 0 \quad (5b)$$

$$A_{33}(t)\dot{\omega}_3 = 0 \quad (5c)$$

thus $\omega_3(t) = s = \text{const}$, whereas

$$A(t)\dot{\alpha} + C(t)\alpha = 0 \quad (6)$$

where $\alpha = \omega_1 + i\omega_2$ and

$$A(t) = I_0 + \sum_{i=0}^{n} m^i[r_3{}^i(t)]^2$$

$$C(t) = 2\sum_{i=0}^{n} m^i r_3{}^i(t)\dot{r}_3{}^i(t) - is\left\{(I_3 - I_0) - \sum_{i=0}^{n} m^i[r_3{}^i(t)]^2\right\}$$

Equation (6) is a simple linear equation and has for its solution

$$\alpha(t) = \alpha(0)\exp\left[-\int_0^t \frac{C(t')}{A(t')}dt'\right] = \alpha_0\exp[\theta(t) + i\phi(t)] \quad (7)$$

which, in components, becomes

$$\omega_1(t) = e^\theta[\omega_1(0)\cos\phi - \omega_2(0)\sin\phi] \quad (8a)$$

$$\omega_2(t) = e^\theta[\omega_1(0)\sin\phi + \omega_2(0)\cos\phi] \quad (8b)$$

From (7), with

$$I^*(t) = I_0 + \sum_{i=0}^{n} m^i[r_3{}^i(t)]^2$$

one has

$$\phi(t) = s\left[I_3\int_0^t \frac{dt'}{I^*(t')} - t\right] \quad (9)$$

$$\theta(t) = -\int_0^t \frac{dI^*(t')}{I^*(t')}$$

so that

$$e^{\theta(t)} = I^*(0)/I^*(t) \quad (10)$$

The integral for ϕ cannot be evaluated until one specifies the motion of the particles. Note that (10) represents a ratio

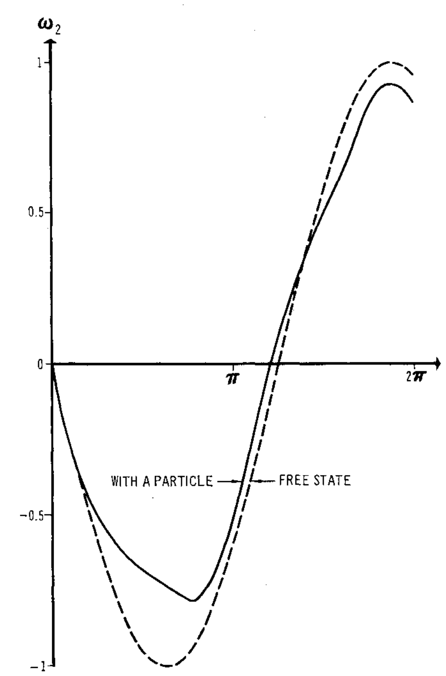

Fig. 2

of longitudinal moments of inertia. From the calculated values of ω_1, ω_2, and ω_3, one can next obtain the attitude at any time by use of the formula for the Hamilton-Cayley-Klein parameters.[2] Since these parameters are given in terms of the history of $\boldsymbol{\omega}(t)$, we have formally integrated the motion for a new class of problems. A last detail of interest is that the coning angle ψ of the vehicle about its angular momentum vector $\mathbf{h}$ is constant; the component of angular velocity normal to the symmetry axis changes inversely in length as the magnitude of the longitudinal moment of inertia (only the coning rate p varies).

Example

Take the case of one particle executing $\beta_3{}^1(t) = a \sin t$, $\beta_3{}^0 = 0$, where $\beta_3{}^1$ is the displacement of the particle from the vehicle's origin, so

$$r_3{}^1 = \beta_3{}^1 - m^1/(m^0 + m^1)\beta_3{}^1 = m^0/(m^0 + m^1)a \sin t$$

$$r_3{}^0 = -m^1/(m^0 + m^1)\beta_3{}^1 = -m^1/(m^0 + m^1)\, a \sin t$$

$$m^0(r_3{}^0)^2 + m^1(r_3{}^1)^2 = m^0m^1/(m^0 + m^1)a^2 \sin^2 t = \mu \sin^2 t$$

From (9) and (10),

$$e^{\theta(t)} = I_0/(I_0 + \mu \sin^2 t)$$

$$\phi(t) = \{I_3 s/[I_0(I_0 + \mu)]^{1/2}\} \tan^{-1}\{[1 + (\mu/I_0)]^{1/2} \tan t\} - st$$

Setting $\alpha = \mu/I_0$, $\beta = (1 + \alpha)^{1/2}$, and $\gamma = I_3/I_0$, we have

$$e^{\theta} = (1 + \alpha \sin^2 t)^{-1}$$

$$\phi = s[(\gamma/\beta) \tan^{-1}(\beta \tan t) - t]$$

With these functional relations, we now use (8) to get the angular velocity. Assume that $a = 15$ ft, $I_0 = 500$ slug ft^2, $I_3 = 100$ slug-ft^2, $m^0 = 20$ slug, $m^1 = 1$ slug, and $s = 1$ rad/sec. A pair of graphs (Figs. 1 and 2) is shown of (8) for the foregoing with $\omega_1(0) = 1$ and $\omega_2(0) = 0$, and a comparison is made with the free-body case.

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Evaporation of Electroplated Cadmium in High Vacuum

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Introduction

Cadmium or zinc platings are often used in missiles as protection against corrosion due to moisture, propellants, and decomposition products of lubricants and propellants.[1] Cadmium-plated articles have also been used in space vehicles. The question has arisen concerning the desirability and feasibility of using cadmium in the space environment.[2] Critics have claimed that the vapor pressure of cadmium is such that it is sublimed and redeposited on critical vehicle areas in the space environment. Proponents of cadmium plating maintain that the volatilization and/or redeposition is not significant. This study was undertaken to determine a few of the properties of electroplated cadmium in a simulated space environment.

Background

The rate of evaporation of an inorganic material in a vacuum is given by the Langmuir equation[3]

$$G = P_v(M/T)^{1/2}/17.14 \qquad (1)$$

where G is the rate of evaporation in grams per (centimeter)^2-seconds, P_v is the vapor pressure of the material in millimeters of mercury, M is the molecular weight of the material in the gas phase, and T is the temperature in degrees Kelvin. Gloria, Stewart, and Savin[4] found that the evaporation of cadmium in a vacuum is described by the Langmuir equation when the vapor pressure P_v is given as[4,5]

$$\log 10 P_v = 8.564 - 5693.1/T \qquad (2)$$

The vapor pressure obtained using this equation is somewhat higher than that cited by Honig.[6] Furthermore, it has been shown for related materials, and indicated for cadmium,[7] that the oxide generally present on plated cadmium decreases the evaporation rate. Thus, the G for plated cadmium articles can be expected to be less than that predicted by Eqs. (1) and (2).

The Langmuir equation may also be used to obtain the rate of condensation if it is modified by multiplying it by the "sticking" coefficient α. Values of α from 0.001 to 0.6 have been obtained on contaminated and relatively clean surfaces.[8,9] Knudsen[10] found that cadmium vapor condensed completely on glass at 78°K. Rapp, Hirth, and Pound[11] obtained values of one for α at high cadmium supersaturation at room temperature on clean metal surfaces.

The attractive forces between the condensed atoms of cadmium and glass are exceptionally weak.[12] Furthermore, Sennett et al.[13] found that a critical flux density must be exceeded for condensation to take place, and this condensation proceeded after the apparently instantaneous appearance of particles having an approximate size of 200°A. Bueche[14] found that the glass must be near liquid air temperature (−233°F) before cadmium could be deposited uniformly. Fraser[15] has suggested that condensation is normal (no critical flux density) if the substrate is perfectly clean, i.e., no adsorbed gases or other contamination.

An interesting phenomenon reported by Palatnik and Gladkikh[16, 17] may prove to be an acceptable explanation of some of the anomalies related to the condensation of metals in vacuums. They found[16] that the transition of the condensation mechanism from vapor → crystal to vapor → liquid (→ crystal) at high vapor concentrations in a vacuum is a "step rule." The second mechanism of condensation was found to begin at a temperature θ_2 (degrees Kelvin), which was given, for the metals studied (including cadmium), by the relationship $\theta_2/T_s = \frac{1}{3}$, where T_s is the melting point of the metal in degrees Kelvin. Furthermore, they found that there is an increasingly broad temperature region, at low vapor concentrations, in which no metal is condensed.[17]

Apparatus and Experimental Procedure

A schematic of the apparatus used for the vaporization and redeposition studies is shown in Fig. 1. The samples were 4 in.^{2}, $\frac{1}{16}$ in., or $\frac{1}{8}$ in. thick, cold-rolled 1020 steel having an electro-

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